A NEW REPRESENTATION OF THE GENERALIZED
RELATIVE DYNAMIC GAINS

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ABSTRACT

A new representation of the generalized relative dynamic gains (GRDG) for input-output pairing of two input-two output (TITO) processes is proposed. This new tool permits a determination of the best specifications for each closed-loop in order to minimize the interaction between the loops when using decentralized control. An application of the new GRDG representation is used to clearly show that the pairing which minimizes the interaction, can depend on the selected specifications. This new tool could be used with aerial systems (missiles, planes, unmanned aerial vehicles, etc.).

Keywords: Interaction; Pairing; Relative gain;

INTRODUCTION

One characteristic of multivariable processes is their interaction. Interaction is an important problem mainly with decentralized control. Several authors have proposed different methods to measure the interaction of multivariable processes\textsuperscript{1,4-19}. Among them, Witcher and McAvoy\textsuperscript{7} and Bristol\textsuperscript{8} have suggested an extension of the well-known relative gain array (RGA)\textsuperscript{6} to the entire frequency domain by defining the relative dynamic gain array (RDGA). This interaction measure is calculated as the RGA but it uses the entire process transfer functions. This method gives the degree of interaction as a function of the frequency. The RGA remains always useful because a negative relative gain leads to unstable or low integrity closed-loop systems with decentralized control\textsuperscript{20}.

Huang and al.\textsuperscript{17} have also developed an interesting dynamic interaction measure. Their method, the generalized relative dynamic gains (GRDG), is an RDGA amelioration. The RDGA is based on the hypothesis that the closed-loop controllers are ideal. It implies that the closed-loop outputs stay constant all the time even if the selected manipulated variable is modified. The GRDG avoid this approximation by including the desired closed-loop dynamics.

In this paper, an extension of the GRDG is presented. The extension consists in calculating each GRDG as a function of the closed-loop dynamics. For a TITO process, the information can be plotted in 3D graphics. The number of 3D graphics obtained can be decreased without loss of useful information by using a scale transformation\textsuperscript{19}. The GRDG extension proposed permits to determine the best dynamics requested for each closed-loop in order to minimize the interaction between the loops when using decentralized control. This new representation which produces more information, improves and simplifies the pairing analysis.

GENERALIZED RELATIVE DYNAMIC GAINS

An interesting interaction measure is the generalized relative dynamic gains (GRDG) of Huang and al.\textsuperscript{17}. It has been previously mentioned that the RDGA makes the hypothesis that the closed-loop controllers are perfect, so that the process outputs stay constant all the time even if the selected manipulated variable is modified. The GRDG avoid this approximation by taking into account the dynamics of the closed-loops.

The GRDG \[ |\xi_{ij}(s)| \] between input (i) and output (j) for a TITO process is defined as follows:

\[
|\xi_{ij}(s)| = \left| \frac{P_{ij}(s)P_{3-i,3-j}(s)\frac{r_{3-i}(s)}{y_{3-i}(s)}}{P_{ij}(s)P_{3-i,3-j}(s)\frac{r_{3-j}(s)}{y_{3-j}(s)} - P_{3-j}(s)P_{3-i,j}(s)} \right| (1)
\]

where \( r_{3,i}(s)/y_{3,i}(s) \) is the desired dynamics of the (3-i)\textsuperscript{th} loop. The variables \( r_{3,i}(s) \) and \( y_{3,i}(s) \) are the (3-i)\textsuperscript{th} set point and the (3-i)\textsuperscript{th} process output respectively. The transfer functions \( P_{ij}(s) \) are the elements of the process transfer matrix \( P(s) \).
The GRDG, as the RDGA and the RGA, do not show the interaction in the case of processes with triangular transfer matrices because they do not measure directly the interaction. In fact, some authors\textsuperscript{5,15,18} have shown that multivariable closed-loop systems can be represented with the transmittances of figure 1.

\[ d_i(s) \]  
\[ \text{Direct} \]  
\[ \text{Parallel} \]  
\[ d_j(s) \]  
\[ \text{Interaction} \]  
\[ y_i(s) \]

Figure 1  Direct, parallel, and interaction transmittances in a multivariable closed-loop system

The GRDG, RDGA and RGA measure the importance of the parallel transmittance in comparison with the direct one. This comparison, even if it is not the best measure of the interaction, is certainly a very good tool for input-output pairing. It is also important to note that the RGA, RDGA and GRDG can indicate if a process is difficult to control\textsuperscript{2,3,21}.

**NEW GRDG REPRESENTATION**

In this paper, a new representation of the GRDG is given for TITO processes. It consists in plotting the GRDG, on 3D graphics, as functions of both closed-loop bandwidths.

Equation (1) shows that the GRDG \[ |\hat{\lambda}_{11}(s)| \] depends on the dynamics of the second loop, \( y_2(s)/r_2(s) \). The dynamics \( y_2(s)/r_2(s) \) is the specification selected by the designer of the control algorithm. Hence, the GRDG \[ |\hat{\lambda}_{11}(s)| \] (z-axis) can be plotted as a function of the frequency (x-axis), for several specifications of the second loop (y-axis), leading to a 3D graphic. The y-axis is graduated with the –3dB bandwidth \( w_{c2} \) of each specification.

For a given value of \( w_{c2} \), the most important frequency to analyze on the x-axis is the one corresponding to the –3dB bandwidth of the first closed-loop\textsuperscript{2,21}. Therefore, the x-axis is labeled as \( w_{c1} \), the bandwidth of the first closed-loop. Similar graphics can be plotted for the GRDG \[ |\hat{\lambda}_{22}(s)|, |\hat{\lambda}_{12}(s)| \] and \[ |\hat{\lambda}_{21}(s)| \].

To facilitate the visualization of the graphics, it is interesting to combine the 3D graphics corresponding to the same input-output pairing, i.e. \[ |\hat{\lambda}_{11}(s)| \] with \[ |\hat{\lambda}_{22}(s)| \] and \[ |\hat{\lambda}_{12}(s)| \] with \[ |\hat{\lambda}_{21}(s)| \]. This can be done by keeping, for each pair, the worst point (the point representing the strongest interaction) between the two graphics for every value of \( w_{c1} \) and \( w_{c2} \). The result is a 3D graphic for each choice of input-output pairing.

However, the worst points are not necessarily the farthest points from unity since the GRDG scale, as the RDGA and the RGA ones, is not linear. A gain of 0.5 indicates the same interaction as a gain of \( \infty \). The scale transformation proposed by Zhu\textsuperscript{19} produces a linear scale and will allow the selection of the worst points. Therefore, the linearized generalized relative dynamic gains (GRDG\textsubscript{L}) are obtained as:

\[
|\hat{\lambda}_{ijL}(s)| = \frac{1}{|\hat{\lambda}_{ij}(s)|} - 1 \tag{2}
\]

where the subscript \( L \) indicates the use of a linear scale. The GRDG and GRDG\textsubscript{L} scales are compared in table 1.

<table>
<thead>
<tr>
<th>GRDG scale</th>
<th>GRDG\textsubscript{L} scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \infty )</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1  The GRDG versus GRDG\textsubscript{L} scales

For each pair of points, the farthest to 0 in the linear scale is selected. The resulting two graphics, one for each input-output pairing, can be used for two related purposes:

- Given specifications \( w_{c1} \) and \( w_{c2} \), select the pairing with GRDG\textsubscript{L} closest to 0.
- Given a process, select the pairing and dynamics \( w_{c1} \) and \( w_{c2} \) in order to obtain minimum interaction.

Input-output pairing is an important problem when designing decentralized control. An improper pairing results in a process very difficult or even impossible to control. The following example uses the new GRDG representation to show that, depending on the specifications, the best pairing to minimize the interaction can be different.
**EXAMPLE**

For the following TITO process:

\[
P(s) = \begin{bmatrix}
4 & -1.33 \\
1+10s & 1+1.1s \\
1+1.1s & 4 \\
\end{bmatrix}
\]  

(3)

The relative gain array is:

\[
RGA = \begin{bmatrix}
0.9 & 0.1 \\
0.1 & 0.9 \\
\end{bmatrix}
\]  

(4)

The \textit{RGA} elements, in equation (4), are all positive. Therefore, both input-output pairings are possible with regard to stability or integrity when decentralized controllers are used. Figure 2 shows the combination of the GRDG \( \lambda_{11L}(s) \) and \( \lambda_{22L}(s) \), and figure 3 the combination of the GRDG \( \lambda_{12L}(s) \) and \( \lambda_{21L}(s) \), when first order dynamics are desired for the closed-loops. In this case, the –3dB bandwidth of each loop, \( w_{c1} \) and \( w_{c2} \), is the inverse of the time constant of the first-order specification. Note that the axis scaling of figure 2 and these of figure 3 are different. This has been done to give a better view of the surfaces.

With the pairing of figure 2, the two closed-loops cannot have, at the same time, a bandwidth approximately larger than 1/10 without increasing interaction between the loops. Only one closed-loop can have a bandwidth approximately larger than 1/10 without increasing the interaction. Figure 3 shows that with the other pairing, the two closed-loops must have a bandwidth approximately larger than one to minimize the interaction between the loops. Therefore, in this example, the best pairing to minimize the interaction depends on the closed-loop specifications.

**CONCLUSION**

A new representation of the GRDG has been proposed for TITO process input-output pairing. This representation permits to determine the best specifications for each closed-loop in order to minimize the interaction between the loops when using decentralized control. This GRDG extension improves and simplifies TITO process input-output pairing selection. An application of the GRDG extension has been used to clearly show that depending on the specifications, the best pairing to minimize the interaction, can be different.

REFERENCES


