SIMPLIFIED, IDEAL OR INVERTED DECOUPLING?

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ABSTRACT

This paper presents a comparative study of simplified, ideal and inverted decoupling. The stability, robustness and implementation of the three decoupling methods are studied. The structured singular value (SSV) is used to carry out some comparisons. It is demonstrated that robust performance and robust stability of a nominally stable control system are equivalent for the three decoupling methods when the controllers are tuned to obtain identical nominal performance. A relation is derived between the presence of right-half plane (RHP) zeros of a process in series with its simplified decoupler and the instability of the ideal and inverted decouplers for the same process. This paper also describes a potential implementation problem related to the particular structure of the inverted decoupling. Finally, a recapitulative table of the main advantages and limitations of each decoupling method is presented.

Keywords: Decoupling; Robustness; Stability; Implementation
1. INTRODUCTION

The choice of a decoupling method is a relatively complex task since all techniques have their advantages and limitations. Simplified decoupling is by far the most popular method. Its main advantage is the simplicity of its elements. Ideal decoupling, which is rarely used in practice, greatly facilitates the tuning of the controller transfer matrix. Inverted decoupling, which is also rarely implemented, presents at the same time the main advantage of both the simplified and ideal decoupling methods.

Some authors have already compared simplified, ideal and inverted decoupling. Luyben [1] and Weischedel and McAvoy [2] have compared ideal and simplified decoupling methods using distillation column simulators. They concluded that simplified decoupling is more robust than ideal decoupling. According to Waller [3], stability problems encountered by Luyben [1] with ideal decoupling are explained by the fact that he used the same controller tuning for both decoupling methods. Weischedel and McAvoy [2] also kept the same controller tuning for both decoupling techniques, therefore leading to the same conclusion about robustness. Following these studies, McAvoy [4] concluded that ideal decoupling is very sensitive to modeling errors.

To evaluate control systems robustness, Arkun and al. [5] proposed a general analysis procedure based on the singular values. To illustrate their methodology, they studied decoupling control systems applied to several distillation columns. The distillation columns used came from the literature [2,6,7]. They compared ideal decoupling and simplified decoupling. However, to be able to carry out direct analysis and comparisons with results already presented, they used the
same decouplers and controllers parameters as found in the literature. As in the preceding studies, they also concluded that ideal decoupling can be less robust than simplified decoupling.

In his book, Shinskey [8] detailed both simplified and inverted decoupling structures. He explains why the initialization problem, which consists in finding the right controller outputs values to allow bumpless switches between the manual and automatic modes, is easier to solve with inverted decoupling technique. He also describes why it is much more easier to take into account saturation of manipulated variables when using inverted decoupling. Furthermore, it can be added that ideal decoupling presents the same deficiencies as simplified decoupling when analyzing initialization and saturation.

Simplified and inverted decoupling methods are also described by Seborg and al. [9]. Referring respectively to Shinskey [8] and to Luyben [1], Waller [3] and Weischedel and McAvoy [2], they concluded that inverted decoupling is appropriate to take into account the saturation of manipulated variables, however it is more sensitive to modeling errors.

Recently, Wade [10] discussed implementation issues for the inverted decoupling method. In most commercial distributed control systems (DCS), the PID function block has an auxiliary input called “feedforward input”. The feedforward input is summed, within the PID function block, with the output of the PID algorithm. Consequently, such a PID function block is appropriate for direct implementation of inverted decouplers, allowing, without any programming, correct initialization, which facilitates bumpless switches between manual and automatic modes.
The purpose of this paper is to show that the three decoupling methods present the same robust stability and robust performance when the controllers are tuned to obtain equal closed loop nominal performance. An example, which uses the structured singular value, depicts this important point. However, the three decoupling methods may not have the same nominal stability. In fact, a relation is established between the presence of right-half plane (RHP) zeros of a process in series with its simplified decoupler and the nominal instability of the ideal and inverted decouplers for the same process. A potential implementation problem with inverted decoupling, which can deteriorate performance, is also explained. It is shown, with an example, that this problem can even destabilize the control system. Finally, this paper summarizes in a simple table the main advantages and limitations of each decoupling method.

2. DECOUPLING METHODS

Decoupling at the input of a two input-two output (TITO) process $P(s)$ requires the design of a transfer matrix $D(s)$, such that $P(s)D(s)$ is a diagonal transfer matrix $T(s)$:

$$
D(s) = \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix}, \quad P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}, \quad T(s) = \begin{bmatrix} T_{11}(s) & 0 \\ 0 & T_{22}(s) \end{bmatrix}
$$

(1)

and

$$
P(s)D(s) = T(s)
$$

(2)

Figure 1 shows a decoupling control system for a TITO process. The variables $r_1$ and $r_2$ are the set points, $c_1$ and $c_2$ are the controller outputs, $u_1$ and $u_2$ are the manipulated variables and $y_1$ and
y_2 are the process outputs. The controller transfer matrix C(s) is diagonal and is defined as follows:

\[
C(s) = \begin{bmatrix}
C_1(s) & 0 \\
0 & C_2(s)
\end{bmatrix}
\tag{3}
\]

Substituting equations (1) into equation (2) leads to:

\[
D(s) = P(s)^{-1}T(s) = \frac{1}{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)} \begin{bmatrix}
P_{22}(s)T_{11}(s) & -P_{12}(s)T_{22}(s) \\
P_{21}(s)T_{11}(s) & P_{11}(s)T_{22}(s)
\end{bmatrix}
\tag{4}
\]

The elements \(P_{11}(s), P_{12}(s), P_{21}(s), P_{22}(s)\) of equation (4), which represent the transfer functions of the process, are supposed to be known. The only unknown elements are \(T_{11}(s)\) and \(T_{22}(s)\). They represent the desired dynamics of the decoupled system.

### 2.1. Ideal decoupling

A first decoupling control design consists in selecting the transfer functions \(T_{11}(s)\) and \(T_{22}(s)\). The decoupling transfer matrix \(D(s)\) is then deduced from equation (4). The diagonal controller elements \(C_1(s)\) and \(C_2(s)\) are independently and respectively tuned based on \(T_{11}(s)\) and \(T_{22}(s)\). A logical choice for \(T(s)\) is \(T_{11}(s) = P_{11}(s)\) and \(T_{22}(s) = P_{22}(s)\). With this choice, the same controller tunings can be kept even if one loop is set in manual mode. However, this technique, called "ideal decoupling" by Luyben [1], often leads to complicated \(D(s)\) expressions, which can be difficult to realize.
2.2. Simplified decoupling

A second decoupling control design, called "simplified decoupling" by Luyben [1], is widely used in the literature. It consists in selecting the decoupler as follows:

\[
D(s) = \begin{bmatrix}
1 & -\frac{P_{12}(s)}{P_{11}(s)} \\
-\frac{P_{21}(s)}{P_{22}(s)} & 1
\end{bmatrix}
\] (5)

The resulting transfer matrix \(T(s)\) is then:

\[
T(s) = \begin{bmatrix}
P_{11}(s) - \frac{P_{12}(s)P_{21}(s)}{P_{22}(s)} & 0 \\
0 & P_{22}(s) - \frac{P_{12}(s)P_{21}(s)}{P_{11}(s)}
\end{bmatrix}
\] (6)

This choice makes the realization of the decoupler easy, but the diagonal transfer matrix \(T(s)\) obtained is complex since its elements are the sum of transfer functions. Controller tuning can therefore be difficult. It is then often suggested to approximate each sum by a simpler transfer function to facilitate controller tuning.

2.3. Inverted decoupling

To avoid the realization problems of ideal decoupling while keeping its advantages, an interesting method, found in Shinskey [8], consists in modifying the decoupling structure of
figure 1. According to this figure and equation (4), when $T_{11}(s) = P_{11}(s)$ and $T_{22}(s) = P_{22}(s)$, the manipulated variables are:

$$u_1(s) = c_1(s) \left[ \frac{P_{11}(s) P_{22}(s)}{P_{11}(s) P_{22}(s) - P_{12}(s) P_{21}(s)} \right] - c_2(s) \left[ \frac{P_{12}(s) P_{22}(s)}{P_{11}(s) P_{22}(s) - P_{12}(s) P_{21}(s)} \right]$$  \hspace{1cm} (7)

$$u_2(s) = -c_1(s) \left[ \frac{P_{21}(s) P_{11}(s)}{P_{11}(s) P_{22}(s) - P_{12}(s) P_{21}(s)} \right] + c_2(s) \left[ \frac{P_{11}(s) P_{22}(s)}{P_{11}(s) P_{22}(s) - P_{12}(s) P_{21}(s)} \right]$$  \hspace{1cm} (8)

It can easily be demonstrated (Appendix A) that these equations can be simplified as follows:

$$u_1(s) = c_1(s) - u_2(s) \frac{P_{12}(s)}{P_{11}(s)}$$ \hspace{1cm} (9)

$$u_2(s) = c_2(s) - u_1(s) \frac{P_{21}(s)}{P_{22}(s)}$$ \hspace{1cm} (10)

Figure 2 depicts a control system with the decoupling structure defined by equations (9) and (10). This representation is called "inverted decoupling" by Wade [10]. It should be noted that the transfer functions of the decoupler are the same as the ones used with simplified decoupling. Therefore, inverted decoupling offers at the same time, the ease of realization of the simplified decoupling elements and the more appropriate diagonal transfer matrix $T(s)$ of the ideal decoupling.
3. ROBUSTNESS AND STABILITY

For equivalent nominal performance, the robust performance and robust stability of nominally stable control systems with simplified, ideal and inverted decoupling are identical. In fact, equivalent nominal performance implies that the closed loop transfer matrices are identical. The process being the same in each case, the controllers transfer matrices in series with the decoupling transfer matrices are also identical for all three decoupling methods. Therefore, they all present the same robustness. However, to obtain the same nominal performance with different decouplers, the controller cannot be the same in all cases.

3.1. Nominal Stability

When analyzing nominal stability, some differences can appear between the three decoupling methods. For some processes, simplified decoupling is nominally stable but ideal and inverted decoupling are unstable. For ideal decoupling, by defining:

\[ P_{11}(s) = \frac{n_{11}(s)}{d_{11}(s)}, P_{12}(s) = \frac{n_{12}(s)}{d_{12}(s)}, P_{21}(s) = \frac{n_{21}(s)}{d_{21}(s)} \text{ et } P_{22}(s) = \frac{n_{22}(s)}{d_{22}(s)} \] (11)

with \( T_{11}(s) = P_{11}(s) \) and \( T_{22}(s) = P_{22}(s) \), the denominator of each element of \( D(s) \) (equation (4)) becomes:

\[ M(s) = n_{11}(s)n_{22}(s)d_{12}(s)d_{21}(s) - d_{11}(s)d_{22}(s)n_{12}(s)n_{21}(s) \] (12)
The transfer matrix $D(s)$ of ideal decoupling is stable if $M(s)$ does not have any root with a positive real part. With simplified decoupling, substituting equation (11) into equation (6) leads to the following expression for the numerator of each element of $T(s)$:

$$N(s) = n_{11}(s)n_{22}(s)d_{12}(s)d_{21}(s) - d_{11}(s)d_{22}(s)n_{12}(s)n_{21}(s)$$  \hspace{1cm} (13)

Since $M(s) = N(s)$, when a process in series with its simplified decoupler has RHP zeros (more difficult controller design), the ideal and inverted decouplers for the same process would be unstable (unstable system).

Therefore, the first step of a robustness study is to verify the nominal stability of the three decoupling methods. Figure 3 shows a closed loop system where $d$ is the process input disturbance vector. $P(s)$ represents the transfer matrix of the nominal process model and $K(s)=D(s)C(s)$ represents the controller in series with the decoupler. Nominal stability of the closed loop system is verified if and only if all elements in transfer matrix $F(s)$ of the following equation:

$$\begin{bmatrix} y(s) \\ u(s) \end{bmatrix} = F(s) \begin{bmatrix} r(s) \\ d(s) \end{bmatrix}$$  \hspace{1cm} (14)
have all their poles in the left-half plane. The matrix $F(s)$ is defined with the sensitivity function $E(s)=[I+P(s)K(s)]^{-1}$, where $I$ is the identity matrix, as:

$$F(s) = \begin{bmatrix} P(s)K(s)E(s) & E(s)P(s) \\ K(s)E(s) & -K(s)E(s)P(s) \end{bmatrix}$$  \hspace{1cm} (15)

### 3.2. Nominal performance

Nominal performance analysis based on the singular values permits to verify if the closed loop system respects the desired performance at the nominal point. This condition [11,12] is:

$$\sigma[E(s)\omega_p(s)] \leq 1 \hspace{0.5cm} \forall \omega$$  \hspace{1cm} (16)

where $\sigma$ is the maximum singular value and $\omega_p(s)$ is the desired performance weight.

### 3.3. Input uncertainty

To evaluate the robustness of a control system, multiplicative input uncertainty can be added to the process input. Multiplicative input uncertainty [12,13] is represented as illustrated in Figure 4, where $\omega_I(s)$ is the relative uncertainty and $\Delta_I(s)$ is the disturbance. From this uncertainty description and the desired performance, robust stability and robust performance conditions can be obtained.
3.4. Robust stability

A robust stability condition, based on the structured singular value, allows to verify if the closed loop system would remain stable in presence of multiplicative input uncertainty. The necessary and sufficient condition [12] is:

\[
\mu[H_I(s)\omega_I(s)] \leq 1 \quad \forall \omega 
\]

where \(\mu\) is the structured singular value and \(H_I(s) = [I + K(s)P(s)]^{-1}K(s)P(s)\) is the input complementary sensitivity function.

3.5. Robust performance

To evaluate if the closed loop system will respect the desired performance even in presence of multiplicative input uncertainty, a robust performance condition must be calculated. A necessary and sufficient condition based on the structured singular value [12] is:

\[
\mu\begin{bmatrix}
-P(s)^{-1}H(s)P(s)\omega_I(s) & -P(s)^{-1}H(s)\omega_P(s) \\
E(s)P(s)\omega_I(s) & E(s)\omega_P(s)
\end{bmatrix} \leq 1 \quad \forall \omega 
\]

where \(H(s) = P(s)K(s)[I + P(s)K(s)]^{-1}\) is the complementary sensitivity function.
3.6. Example

It is possible to compare the robustness of the three decoupling methods using a simple process given by:

\[
P(s) = \begin{bmatrix}
\frac{4}{1+10s} & \frac{3}{1+10s} \\
\frac{3}{1+10s} & \frac{4}{1+10s}
\end{bmatrix}
\]  \hspace{1cm} (19)

For this process, decoupling systems will be symmetrical and both non-zero elements of the controllers will be identical. The nominal closed loop performance required for the three systems studied is described by a first order dynamics with a time constant of five time units. PI controllers will be sufficient to reach the desired performance. With simplified decoupling, the matrices \(D(s)\) and \(C(s)\) required respectively to perform decoupling and to obtain the desired nominal performance are:

\[
D(s) = \begin{bmatrix}
1 & -\frac{3}{4} \\
-\frac{3}{4} & 1
\end{bmatrix} \quad \text{and} \quad C(s) = \begin{bmatrix}
\frac{8(1+10s)}{7} & 0 \\
\frac{10s}{10s} & \frac{8(1+10s)}{7}
\end{bmatrix}
\]  \hspace{1cm} (20)

For this example, an IMC-based tuning method [12] has been used. Any other method leading to specific closed-loop dynamics could have also been used. When designing ideal decoupling, the required transfer matrices \(D(s)\) and \(C(s)\) are:
The two transfer functions of the inverted decoupler are \(-3/4\) and the controller \(C(s)\) is the same as with ideal decoupling. Figure 5 shows the poles locus of the transfer matrix \([13] F(s)\) given by (15), for all three decoupling methods. Obviously, they are the same in all cases. This figure points out that the three systems are nominally stable. To analyze robustness, an arbitrary uncertainty of 30% on each process input is selected:

\[
D(s) = \begin{bmatrix} 16/7 & -12/7 \\ -12/7 & 16/7 \end{bmatrix} \quad \text{and} \quad C(s) = \begin{bmatrix} (1+10s)/2 & 0 \\ 10s & (1+10s)/2 \\ 0 & 10s \end{bmatrix}
\]

\( (21) \)

\[\omega_i(s) = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix} \quad \text{and} \quad \Delta_i(s) = \begin{bmatrix} \Delta_1(s) & 0 \\ 0 & \Delta_2(s) \end{bmatrix} \]

\( (22) \)

It is important to precise that an equal uncertainty over all frequencies is not realistic since modeling errors are generally larger at high frequencies than at low frequencies. In order to verify the specifications, \(\omega_p(s)\) has to be defined as follows:

\[
\omega_p(s) = \begin{bmatrix} \frac{5s+1}{5s} & 0 \\ \frac{5s}{5s} & \frac{5s+1}{5s} \end{bmatrix}
\]

\( (23) \)

Figure 6 shows the robust stability, nominal performance and robust performance conditions for the three decoupling methods. All three decoupling methods lead to identical results. The nominal performance is described by a unitary magnitude over the whole frequency window.
Therefore, the systems exactly respect the nominal specifications. The robust stability magnitude is clearly smaller than one for all frequencies, indicating that the systems will remain stable in spite of an uncertainty of 30% on each process input. The robust performance analysis shows that the performance will deteriorate at low frequencies, but in the same way for the three systems.

Hence, from a robustness point of view, the three decoupling methods are equivalent when the controllers are tuned to obtain identical closed loop nominal performance. Other types of uncertainty would lead to the same conclusion. In all cases, to obtain the same robustness, closed loop nominal performance must also be perfectly identical. For processes with more complex dynamics (with dead times for instance), it can be very difficult, even impossible, to tune the controllers to obtain identical nominal performance for the three decoupling methods. For some processes, it may also happen that the elements of the decouplers cannot be perfectly realized and some differences may then appear in the robustness analysis.

4. IMPLEMENTATION

When a system is nominally stable with ideal or inverted decoupler, it is as robust as a simplified decoupled system. Since the robustness study deals with process variations and not with controller or decoupler variations, it is independent of the decoupler proximity to instability. In practice, however, it is recommended to make sure that some variations to the controller and decoupler parameters do not lead to instability [14]. In fact, slight differences between calculated and implemented parameters may appear due to the number of significant digits. Fine-tuning of the controller and decoupler must also be possible without destabilizing the system. Finally, an
unwanted delay can be created in one of the decoupling elements, when implementing inverted decoupling on a TITO process using lead-lag and delay function blocks.

4.1. Antireset windup

In most commercial DCS, the PID function block can prevent reset windup through the use of a reset feedback input. The signal sent to the reset feedback input should be equal to the PID output, unless there is saturation. In this case, the reset feedback is the saturated PID output. The antireset windup feature of the PID, combined to its feedforward input, can be used to directly take into account the saturation of the manipulated variables when an inverted decoupling structure is implemented. This is shown for a TITO process in figure 7.

The antireset windup feature is more difficult to implement with simplified and ideal decoupling. The same difficulty arises with inverted decoupling when the PID function bloc does not have feedforward input. Figure 8 shows a decoupling control system (ideal or simplified) where the antireset windup features of the PID blocks are used. This figure shows that it may be difficult to use the antireset windup feature with ideal decoupling due to the impossibility, in some cases, to invert the main diagonal elements of the decoupling transfer matrix.

4.2. Manual mode

Occasionally, for maintenance or any other reason, it may be necessary to operate one loop in manual while the other one remains in automatic mode. With inverted decoupling, the system
remains decoupled when the manual manipulated variable is modified, which is not the case for ideal or simplified decoupling.

### 4.3. Initialization

The decoupling control system must allow bumpless switches between manual and automatic modes. Therefore, before switching from the manual to the automatic mode, the controller outputs must be back calculated from the manipulated variables in manual mode. Unfortunately, since the decoupling elements are often not static, their outputs will not instantaneously take the right values, unless the lead-lag and delay blocks can be initialized to a specific value.

The initialization problem can be avoided by waiting the decoupler to be in steady state before switching to the automatic mode. This limitation is not present with inverted decoupling because the input of each decoupling element is a manipulated variable measurement.

### 4.4. Unwanted delay

Because of the blocks scanning sequence, when implementing inverted decoupling for a TITO process on a DCS using lead-lag and delay function blocks, an unwanted delay can be created in one of the decoupling elements. This delay of one sampling period slightly decreases decoupling system performance. With some processes, it can even destabilize the control system if the calculated decoupler was close to instability, as illustrated by the following example.
4.5. Example

The TITO process is defined as follows:

\[
P(s) = \begin{bmatrix} \frac{3}{1+15s} & 4 \\ 1+10s & \frac{3}{1+15s} \end{bmatrix}
\]  

(24)

As illustrated in figure 2, the transfer functions of the inverted decoupler are:

\[
-\frac{P_{12}(s)}{P_{11}(s)} = -\frac{P_{21}(s)}{P_{22}(s)} = -\frac{4(1+15s)}{3(1+10s)}
\]  

(25)

The resulting transfer matrix \(T(s)\) is therefore:

\[
T(s) = \begin{bmatrix} \frac{3}{1+15s} & 0 \\ 0 & \frac{3}{1+15s} \end{bmatrix}
\]  

(26)

To reach the nominal performance defined by a first order dynamics with a time constant of five time units, the controller is:

\[
C(s) = \begin{bmatrix} \frac{1+15s}{15s} & 0 \\ \frac{15s}{0} & \frac{1+15s}{15s} \end{bmatrix}
\]  

(27)

The next step in the design consists in verifying nominal stability. The process is obviously stable. The inverted decoupler, equivalent to \(D(s)\) defined by equation (4) with \(T_{11}(s) = P_{11}(s)\) and
$T_{22}(s)=P_{22}(s)$, is also stable. Figure 9 shows the poles locus of the equivalent decoupling transfer matrix $D(s)$. The nominal stability of the closed loop system is verified, if the closed loop transfer matrix $F(s)$ does not have any poles with a positive real part. Figure 10 shows the nominal stability of the closed loop system.

Figure 11 shows the nominal performance, robust stability and robust performance conditions for an uncertainty of 30% on each process input. The nominal performance is respected. The system remains stable in spite of the uncertainty and the performance deterioration is more important at low frequencies. However, the system becomes unstable if the implementation of this control system with function blocks introduces a delay of one scan time (1 time unit in this case) in one of the decoupler elements. Figure 12 shows this instability by using a Pade approximation for the unwanted delay added to the closed loop transfer matrix $F(s)$. Figure 13, which shows the poles locus of the equivalent transfer matrix $D(s)$ of the inverted decoupler with the unwanted delay, indicates that the instability comes from the decoupler.

It is therefore very important to take into account a possible delay introduced by the blocks scanning sequence. To avoid the introduction of the delay, inverted decoupler could be implemented using a unique multivariable block (possibly using a state-space representation). Also, if the inverted decoupling cross-elements require dead times due to the delays of the process, the designer can subtract a delay of one scan time in one cross-element of the inverted decoupling structure at the implementation time. However, the designer must be careful to the order in which the chains of blocs are executed in the control system, to really compensate for the artificial delay and to avoid the amplification of the problem.
5. SUMMARY TABLE

Table 1 summarizes the advantages and disadvantages of each decoupling method. The robustness is not discussed since it is identical for all three decoupling techniques.

<table>
<thead>
<tr>
<th>Decoupling methods Characteristics</th>
<th>Simplified decoupling</th>
<th>Ideal decoupling</th>
<th>Inverted decoupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>When one loop is in manual mode, dynamics of the remaining loop is unchanged.</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Decoupling elements do not contain a sum of transfer functions.</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Transfer matrix of the decoupler in series with the process does not contain sums of transfer functions.</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>When loops are switched from manual to automatic mode, decoupling system initialization is simple.</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Saturation of the manipulated variables is easily taken into account with a PID function block having an antireset feature and a feedforward input.</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Implementation with lead-lag and delay function blocks may not decrease performance.</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>A feedforward input to the PID function block facilitates the decoupling system implementation.</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>The antireset windup of the PID function block can be used without inverting a transfer function.</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
</tr>
</tbody>
</table>

Table 1 Advantages and disadvantages of each decoupling method
6. CONCLUSION

This paper gives some guidelines for the selection of a decoupler. It is shown that robust performance and robust stability of nominally stable control systems, using simplified, ideal and inverted techniques, are the same if the controllers are tuned to obtain the same nominal performance. Therefore, the selection of one of the three methods must not be based on robustness considerations. A relation has also been established between the presence of RHP zeros of a process in series with its simplified decoupler and the nominal instability of the ideal and inverted decouplers for the same process. It is also shown that inverted decoupling performance can depend on the implementation method. Finally, a table summarizes the main advantages and limitations of each method.

REFERENCES


APPENDICE A

Demonstration

\[ u_1(s) = c_1(s) \left[ \frac{P_{11}(s)P_{22}(s)}{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)} \right] - c_2(s) \left[ \frac{P_{12}(s)P_{22}(s)}{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)} \right] \]  

(A.1)

\[ u_2(s) = -c_1(s) \left[ \frac{P_{11}(s)P_{22}(s)}{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)} \right] + c_2(s) \left[ \frac{P_{11}(s)P_{22}(s)}{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)} \right] \]  

(A.2)

Therefore,

\[ u_1(s) \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] = c_1(s) - c_2(s) \frac{P_{12}(s)}{P_{11}(s)} \]  

(A.3)

and

\[ u_2(s) \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] = c_2(s) - c_1(s) \frac{P_{21}(s)}{P_{22}(s)} \]  

(A.4)

According to equation (A.3)

\[ c_1(s) = u_1(s) \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] + c_2(s) \frac{P_{12}(s)}{P_{11}(s)} \]  

(A.5)

According to equation (A.4)

\[ c_2(s) = u_2(s) \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] + c_1(s) \frac{P_{21}(s)}{P_{22}(s)} \]  

(A.6)
Substituting equation (A.6) to equation (A.3) leads to

\[ u_1(s) \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] = c_1(s) \]

\[ -u_2(s) \frac{P_{12}(s)}{P_{11}(s)} \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] - c_1(s) \frac{P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \]  \hfill (A.7)

Substituting equation (A.5) to equation (A.4) leads to

\[ u_2(s) \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] = c_2(s) \]

\[ -u_1(s) \frac{P_{21}(s)}{P_{22}(s)} \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] - c_2(s) \frac{P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \]  \hfill (A.8)

Therefore,

\[ u_1(s) \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] = c_1(s) \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] \]

\[ -u_2(s) \frac{P_{12}(s)}{P_{11}(s)} \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] \]  \hfill (A.9)

and

\[ u_2(s) \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] = c_2(s) \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] \]

\[ -u_1(s) \frac{P_{21}(s)}{P_{22}(s)} \left[ \frac{P_{11}(s)P_{22}(s) - P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)} \right] \]  \hfill (A.10)

Finally,

\[ u_1(s) = c_1(s) - u_2(s) \frac{P_{12}(s)}{P_{11}(s)} \]  \hfill (A.11)

and

\[ u_2(s) = c_2(s) - u_1(s) \frac{P_{21}(s)}{P_{22}(s)} \]  \hfill (A.12)
FIGURE LEGENDS

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FIGURES

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Figure 2
Robust Stability
Nominal performance
Robust performance

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