A study of turbo codes for multilevel modulations in Gaussian and mobile channels

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ABSTRACT - In this paper, we present simulation results for a system combining turbo codes and multilevel modulation. This scheme is shown to be an excellent solution when it comes to transmission over bandlimited channels. Its performance in mobile radio channels can be improved using OFDM.

1. Introduction

Since the first publications regarding turbo codes, various studies have been carried out. Most of these studies illustrated performances or dealt with the optimization of the structure of turbo codes using binary modulation. The requirement for higher information transmission rates on bandlimited channels have led some researchers to consider turbo codes with spectrally efficient modulations.

In [2], an application of turbo codes to trellis coded modulation (TCM) called parallel concatenated trellis coded modulation is presented. At the encoder this scheme uses two recursive systematic convolutional coders (RSC) of rate $R = b/(b+1)$, b being even, and two interleavers for a throughput of b bits/sec/Hz with 2^{2b+2}-QAM. The decoding is performed iteratively by the symbol MAP decoder followed by a bit reliability calculation module.

In [3], a turbo encoder employing two Ungerboeck codes in their recursive form as component codes has been used in combination with TCM. This turbo TCM (TTCM) uses an interleaver operating over symbols of *2* bits for an 8 PSK modulation. The variables passed between symbol MAP decoders are vectors of four log-likelihood ratios (LLRs), one for each possible information group value.

These two models have in common a major modification on the classical turbo encoder to fit the TCM model at the expense of an increase in complexity at the decoder. This paper presents some results obtained with turbo codes and multilevel modulations in Gaussian and mobile radio channels by applying the so called "pragmatic" approach [1]. Section 2 deals with the encoder structure and the signal mapping scheme. Section 3 presents the channels models used in the simulations and the Section 4 presents the decoding operation. The last two sections are devoted to the simulations results and some concluding remarks.

2. Coding and modulation schemes

The encoder is made up of two RSCs with constraint length k = 3 and polynomial generators (5,7). The RSCs are separated by an uniform block interleaver. For short frame transmission applications, the interleaver was set to a size of 192 bits which is compatible with the IS-95 CDMA cellular standard. Another block size of 1022 bits has been used. This size, after adding 2 bits for terminating the RSCs, match a COFDM symbol with binary modulation. The output of the 1/3 turbo encoder is punctured to obtain a 1/2 coding rate.

The system presented here is different from the TCM approach in the sense that it uses a coding rate R = 1/2 for a 16 point constellation. The output sequence of the encoder $(u_1, c_1, u_2, c_2, ...)$ is modulated by two 4-ASK in quadrature resulting in a 16-QAM system where the I and Q components are independently coded (using Gray code). The ASK symbols $\{A_k, B_k\}$ are then transmitted through the channel.

3. Channel models

Generally, for purpose of simulation, two approaches are used to model a mobile radio channel. The first one deals with the multipath phenomenon and the second one deals, in addition to multipath, with frequency selectivity [7]. A Rayleigh flat fading channel has been used to simulate the multipath phenomenon. In this case, the received symbol, (x_k, y_k), is given by

$$\begin{aligned} x_k &= \alpha_k \cdot A_k + i_k \\ y_k &= \beta_k \cdot B_k + q_k \end{aligned} \tag{1}$$

where (α_k, β_k) are two independent Rayleigh distributed random variables and (i_k, q_k) are the additive white Gaussian noise components with variance σ^2.

The frequency selectivity has been introduced by means of intersymbol interference (ISI). In an air-to-ground

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communication, the channel is characterized by a direct line of sight (LOS) and a multipath component with a relative delay [5]. This delayed path creates some ISI on the received signal

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with n representing the relative delay of the diffuse component. This model fits the Rician fading channel.

For applications in microwave LOS radio channels, a similar model has been considered, the Rummler channel which is a two path model [6]

$$H(f) = a(1 - b \cdot e^{-j2\pi(f-f_0)\tau_0}) \tag{3}$$

where a is the overall attenuation, and the term within parenthesis represents the interference between two rays having a relative delay τ_0 and producing a minimum amplitude at the notch frequency f_0. b is the relative amplitude of the multipath ray.

The outputs of the channel are coherently demodulated and the soft inputs of the decoder are then (X_k, Y_k).

4. Decoder structure

The decoding of this multilevel modulation system is much simplified by use of the "pragmatic" approach [1]. The turbo decoder, implemented with the SOVA [4] algorithm, and optimized for binary modulation, can then be used. With this approach, the decoder must be preceded by a module that computes the LLR of each bit contained in an ASK symbol. The LLR can be expressed as follows:

$$\Lambda(u_{k,i}) = \frac{\sigma^2}{2} \cdot \log\left(\frac{\sum_{i=1}^{2} \exp-\left(\frac{1}{2\sigma^2} \cdot (X_k - a_{0,i})^2\right)}{\sum_{i=1}^{2} \exp-\left(\frac{1}{2\sigma^2} \cdot (X_k - a_{1,i})^2\right)} \right) \tag{4}$$

with $u_{k,1} = u_k$ and $u_{k,2} = c_k$ and $(a_{1,i}, a_{0,i})$ are the realizations of symbol A_k conditional to $u_{k,i} = 1$ and $u_{k,i} = 0$ respectively. A good approximation of this expression is given by

$$\begin{aligned} \Lambda(c_k) &= |X_k| - 2 \\ \Lambda(u_k) &= X_k \end{aligned} \tag{5}$$

Although this approximation can lead to good performance, the expression of $\Lambda(u_k)$ in (5) is an overestimation of its real value. A better approximation can be obtained by defining the bit-mapping in one dimension of the constellation with unit energy as follows

$$\begin{aligned} A_k &= 2d \cdot (1 - 2u_{k,1}) \\ &\quad + d \cdot (1 - 2u_{k,2}) \cdot \mathrm{sign}(1 - 2u_{k,1}) \end{aligned} \tag{6}$$

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$\Lambda(u_k)$ can now be approximated by the first term on the second part of this equation. Figure 1 show three curves of $\Lambda(u_k)$ versus X_k. The solid line represents the true expression in (4), the "*" line represents the first approximation in (5) and the "o" line the last approximation. The approximation in (7) is closer to its real value and as it will be shown later, leads to a better performance.

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The simulations have been carried out using the Signal Processing WorkSystem (SPW) package. The BER was computed after 3 iterations of the decoder.

Figure 2 shows for a Gaussian channel 3 BER curves. These curves demonstrate that turbo codes combined with 16-QAM (TC-QAM), even with short frame size and non random block interleaver, give a better performance than conventional TCM with the same throughput. The LLRs were computed as in (5). The coding gain over the TCM can be increased by using the second approximation in (7). In fact, by using this equation to compute the LLR, the performance of the TC-QAM can be improved as is shown in Figure 3.

For the slow Rayleigh flat fading channel, the results with QPSK modulation appear on Figure 4. A BER of 10^{-5} at a SNR around 7.5 dB is achieved by the TC.

On the channels with ISI, we intended to compare the turbo COFDM (T-COFDM) with the TC-QAM. On the Rice model, the multipath component was delayed by an amount of 10 µsec. relative to the LOS path and the carrier to multipath ratio C/M was set to 10 dB. A frame size of 1022 bits was used. The T-COFDM has a coding gain of 1db over the TC-QAM at a BER of 10^{-4} (Figure 5).

On the Rummler channel the relative delay between the two paths was set to 16 µsec. and the attenuation factor, *b*, set to 0.2. In this case also, the T-COFDM outperforms the TC-QAM; the BER curve is shown in Figure 6.

6. Conclusion

In this paper we presented a study of turbo codes combined with multilevel modulations. The system described here could be an alternative to the conventional TCM and is less complex than the PCTCM and TTCM. In frequency selective channels, a combination of TC with OFDM results in a power and spectrally efficient system.

7. References

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7. W. C. Lee, "Mobile Communication Engineering", McGraw-Hill, 1982.

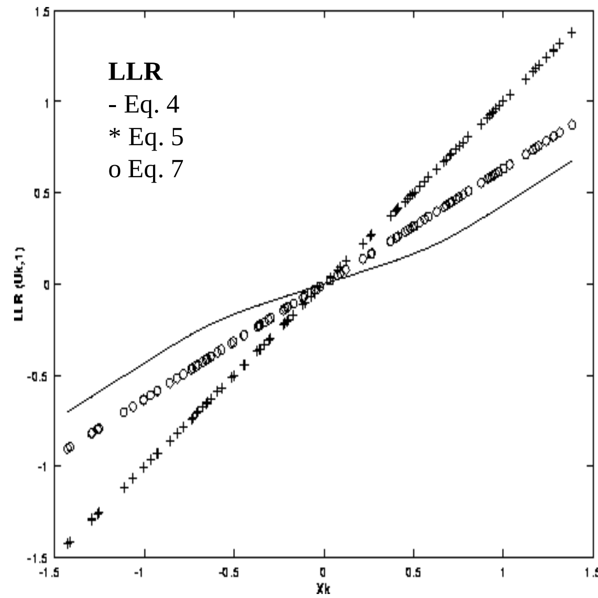


Figure 1: $\Lambda(u_k)$ versus X_k.

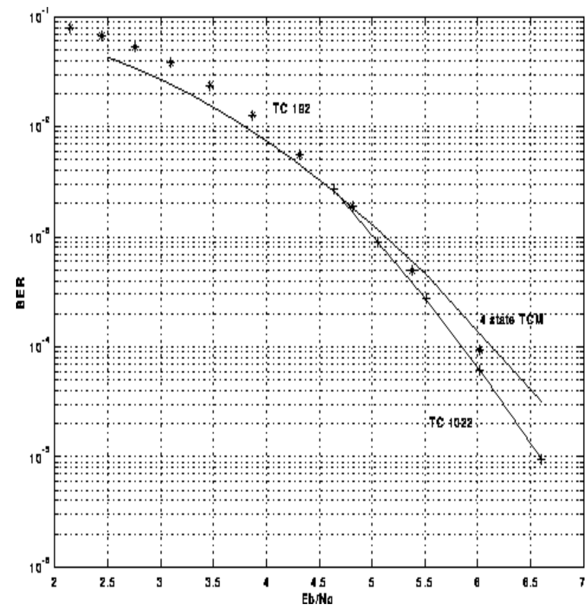
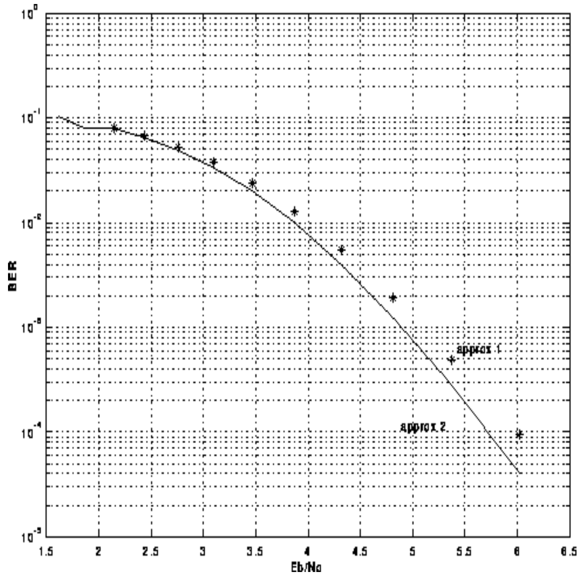


Figure 2: Comparison of TC-QAM with TCM.



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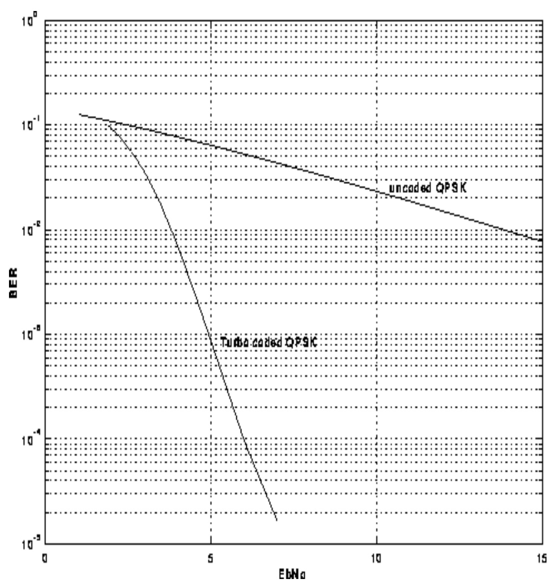

Figure 4: BER curves of binary TC and uncoded QPSK on a Rayleigh channel.

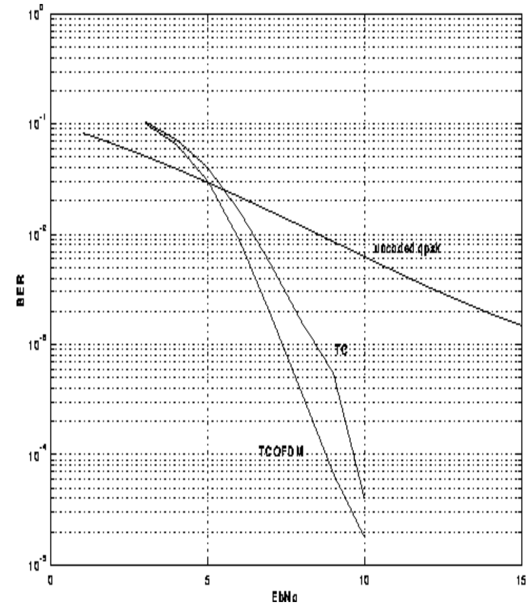

Figure 5: BER curves of T-COFDM and TC-QAM on a Rician channel with ISI.

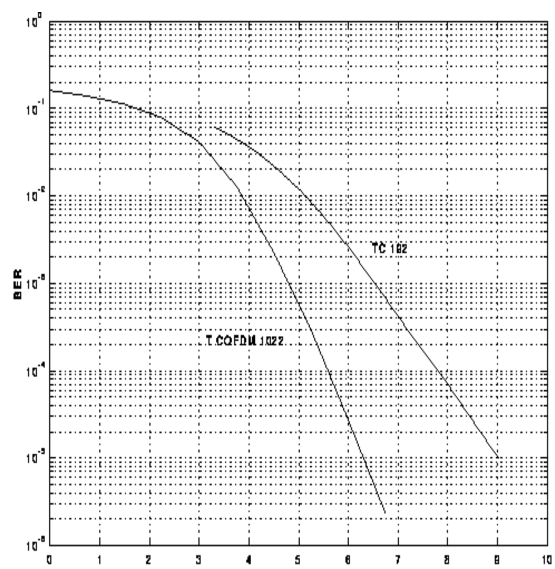

Figure 6: BER curves of T-COFDM and TC-QAM on a Rummler channel.

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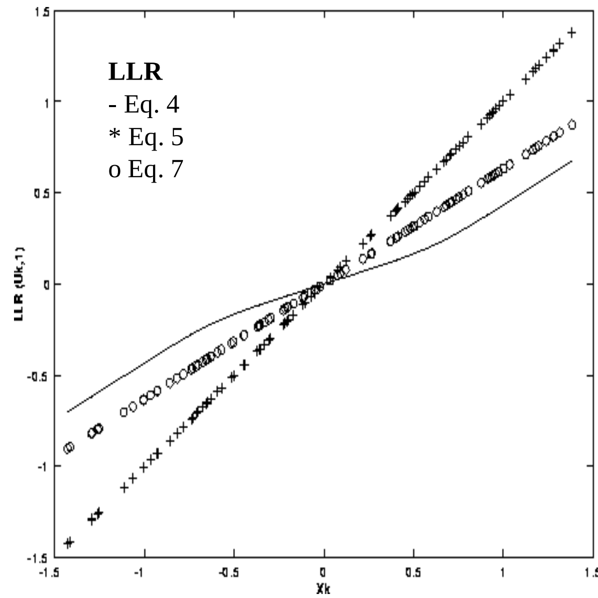

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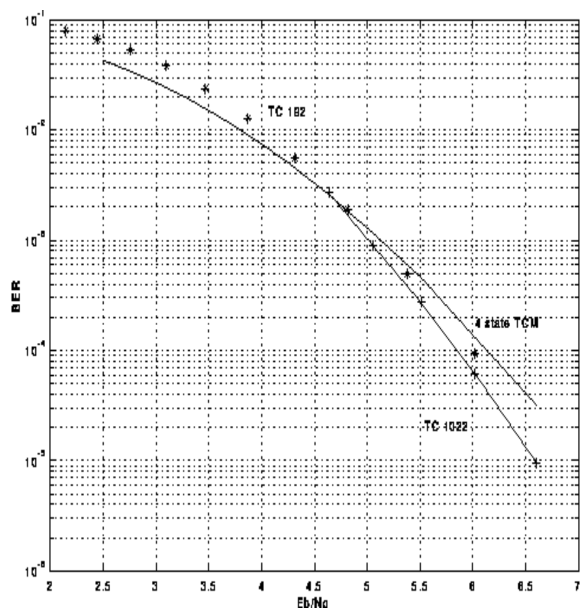

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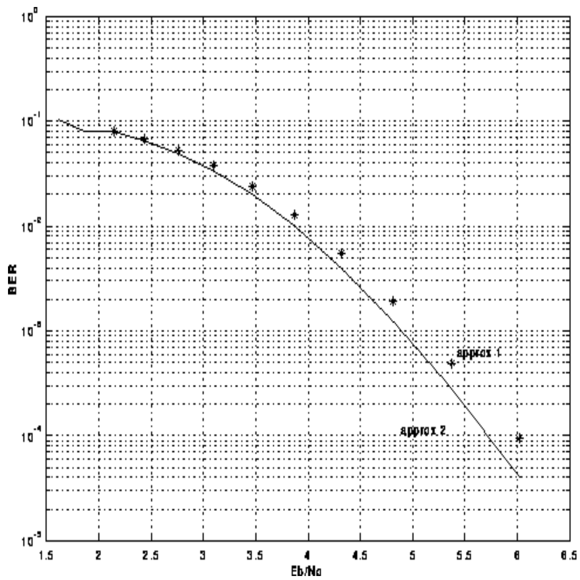

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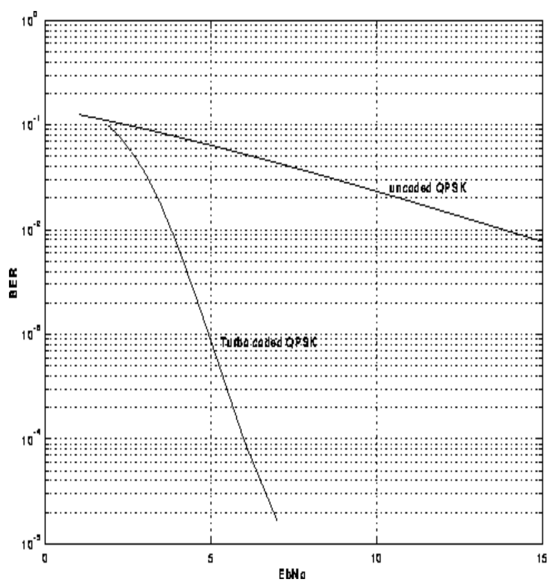

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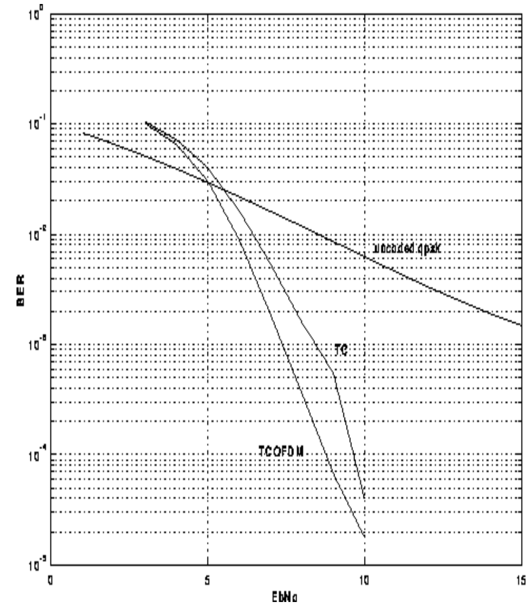

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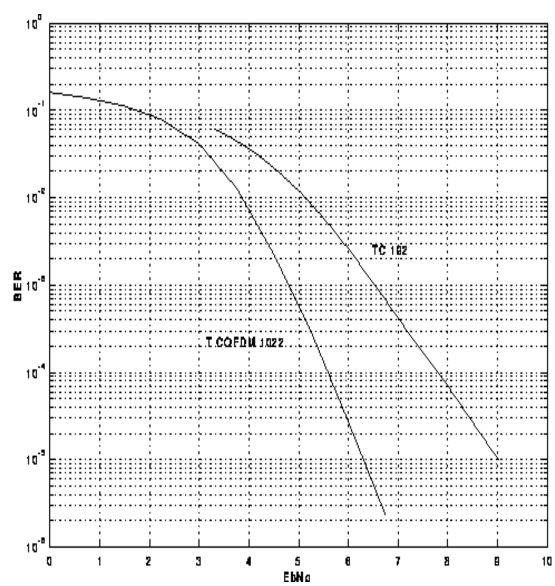

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