Maximum Entropy Random Field Model for Texture Analysis

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Abstract

It is well known that stochastic models, such as MRF and AR model, suffer from their poor capability to describe complicated textures. By introducing filters into these random field models appears to be a new approach to overcome the problem. In this paper, we present a new model for texture analysis, which integrates filter theory into random field based on Maximum Entropy theory. And parameter estimation employing Markov Chain Monte Carlo (MCMC) is proposed for this new model.

Key words: Maximum entropy, texture modeling

1 Introduction

AR model and MRF model have been applied to the field of texture research many years ago[1], [2], [3], [4]. The randomness in texture is the main reason why people consider random field as one of possible ways to model and analyze texture images. But these random field models seem not adequate to represent the texture image effectively.

Intuitively, texture seems to stand between noise and signal. If we choose random field as texture model, then our model is likely close to noise and far from signal. Actually, when facing the task of shaping random field into texture models, people have difficulty in imposing proper constrains on random fields to bring the structure information into texture models.

The FRAME model, proposed by Zhu [5], introduces filters into random field models to extract texture features. This idea combines filtering theory and Markov random field modeling through the maximum entropy principle. With FRAME model, Zhu has tried to interpret and clarify many previous concepts and methods for texture analysis and synthesis from a unified point of view[6].

In this paper, we propose a different model of integrating filter theory with random field. The new model is called maximum entropy random field model. In this texture model, we use filter to directly describe the contextual constraints in texture. Maximum entropy method was employed to construct the joint probability distribution for texture image. For the purpose of fitting this maximum entropy random field model to natural texture, we propose a parameter estimation method based on MCMC.

This paper is organized as follows. Our new texture model will be explained in section 2, its MCMC parameter estimation method in section 3 and experiment results in section 4. Section 5 is conclusion and future work.

2 Maximum Entropy Random Field Model

The objective of random field texture modeling is to find the contextual constraints in texture and describe it. Maximum entropy random field model proposed in this paper assumes the contextual constraint, and employs maximum entropy method to construct the joint probability distribution for texture image.

The reason for the choose of maximum entropy method (MEM) is that MEM suggests one joint probability which has the greatest entropy after meeting given constraints. So this joint probability distribution means to satisfy the constraints along some dimensions, and at the same time, to be as random as possible in other unconstrained dimensions, for entropy is a measure of randomness.

2.1 Maximum Entropy Method

Maximum entropy (ME) is an important principle in statistics for constructing a probability distribution p on a set of random variables X. Suppose the available information is the expectations of some known functions $\phi_n(x)$, i.e.,

$$E_p[\phi_n(x)] = \int \phi_n(x)p(x)dx = \mu_n, \text{for } n = 1, \cdots, N. \tag{1}$$

Let Ω be the set of all probability distribution $p(x)$ which satisfy the constraints, i.e.,

$$\Omega = \{p(x)|E_p[\phi_n(x)] = \mu_n, n = 1, \cdots, N\}. \tag{2}$$

The ME principle suggests that a good choice of the probability distribution is the one that has the Maximum entropy, i.e.,

$$p^*(x) = argmax\{-\int p(x)\log p(x)dx\}, \tag{3}$$

subject to

$$E^p[\phi_n(x)] = \int \phi_n(x)p(x)dx = \mu_n, n = 1, \cdots, N, \tag{4}$$

and

$$\int p(x)dx = 1. \tag{5}$$

By using Lagrange multipliers, the solution for $p(x)$ is

$$p(x;\Lambda) = \frac{1}{Z(\Lambda)}\exp\{-\sum_{n=1}^{N}\lambda_n\phi_n(x)\} \tag{6}$$

where $\Lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ is the Lagrange parameter, and

$$Z(\Lambda) = \int \exp\{-\sum_{n=1}^{N}\lambda_n\phi_n(x)\}dx$$

is the partition function that depends on Λ.

2.2 Deriving Maximum Entropy Random Field Model

Maximum entropy random field model is proposed on the base of an assumption about contextual constraints for texture images. This assumption can be expressed as follow:

There is one particular filter f_n, to which the response of one observed texture has its pixel follow one particular probability distribution. And because the characteristics of homogeneous for observed texture is assumed, so this probability distribution is taken as being equal to the histogram of this response image. This constraints can be written as

$$E\{\delta(f_n(N) - y_j)\} = p(y_j)$$

$$f_n(N) = \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4 \tag{7}$$

where $p(y_j)$ is the histogram of response image, N represent every small lattice on observed texture image, with the same size as that of filter mask f_n. Here we taken the size of filter mask as 2×2.

Based on these constraints, we obtain the following joint probability distribution for every small lattice N in observed texture using Maximum Entropy method.

$$\begin{aligned} f(N) &= \exp\left(-\sum_{k\in N}\lambda_k\delta(f_n(N) - y_k)\right) \\ &= \exp\left(-\lambda(f_n(N))\right) \end{aligned} \tag{8}$$

where $\lambda()$ is function of Lagrange parameters, and it is calculated according to the histogram of response image.

In the case that no other constraints are imposed on observed texture, by following the form of joint probability distribution for Markov random field, we get the joint probability distribution for observed texture as

$$\begin{aligned} f(\mathbf{I}) &= \exp\left(-\sum_{x=1}^{M}\sum_{y=1}^{N}\lambda(f_n(N_{x,y}))\right) \\ &= \exp\left(-\sum\sum\lambda(f_n * \mathbf{I})\right) \end{aligned} \tag{9}$$

where I is the texture image, the sum is carried out on all small lattices across the observed texture.

The above equation is to be used by maximum entropy random field model to describe texture pattern. The function of λ is chosen by experience as the shape showed in Figure 1.

And there are experiment results indicating that the response of very different image to a wide variety of filters has quite similar histogram, so the shape of λ can be taken as the same.

There are two important features of this maximum entropy random field model:

- Because the shape of λ is fixed, so the feature of observed texture depends on filter f_n to describe, and this filter need to be determined for one particular observed texture through process of parameter estimation.

- The parameters for filter f_n is actually the feature of observed texture. These parameters can be used for texture classification and segmentation.

3 Parameter Estimation Using Markov Chain Monte Carlo

In maximum entropy random field model, it is the filter that determines features of texture, so this particular filter is the parameter required to be estimated from one given texture images.

In the field of parameters estimation, Bayesian method receives the most of recognition for a long time. But, in many case, we have great difficulty in carrying out parameter estimation with Bayesian method. Markov Chain Monte Carlo is a new approach to facilitate Bayesian parameter estimation [7], [8], [9], [10].

From Bayesian inference, the joint probability distribution $p(I, \theta)$ comprises two parts, a prior distribution $P(\theta)$ and a likelihood $P(I|\theta)$. Specifying $P(\theta)$ and $P(I|\theta)$ gives a full probability model, in which

$$P(I, \theta) = P(I|\theta)P(\theta) \tag{10}$$

According to Bayesian theory, the posterior distribution of θ conditional on I is

$$P(\theta|I) = \frac{\int P(\theta)P(I|\theta)}{\int P(\theta)P(I|\theta)d\theta} \propto P(\theta)P(I|\theta) \tag{11}$$

For parameter estimation by way of Markov chain Monte Carlo, we need to find out the conditional posterior distribution for every parameter. This is the most important part in the whole MCMC parameter estimation.

Assume there is no prior on the distribution of θ, so likelihood is used to represent the posterior distribution of $P(\theta|I)$,

$$P(\theta|I) \propto \exp \quad -\sum_{x=1}^{M}\sum_{y=1}^{N} \lambda(f_n(N_{x,y}))\Big)$$

$$= \exp\left(-\sum\sum \lambda(f_n * \mathbf{I})\right) \tag{12}$$

The expression for conditional posterior distribution of every parameter is difficult to obtain due to the complexity of the function λ. In our computation, we make the following simplified disposing for approximation.

In equation (17), after taking away the operation of sum across the whole texture image, we get

$$\exp(-\lambda(\theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4)) \tag{13}$$

This equation can be considered as probability distribution of one parameter in f_n, it has pdf similar to the form showed in Figure 2. The real conditional posterior distribution for this parameter is equal to the product of probability distribution like equation (18) across all lattice in the whole texture image.

For simplicity, we take one exponential distribution to approximate the shape of the real conditional posterior distribution for every parameter in f_n. But this exponential distribution is taken as of symmetry. The symmetrical point p_0 is taken as the central point in terms of weight.

For example, the conditional posterior distribution for parameter θ_1 is approximated by one symmetrical distribution with one half like exponential distribution. And the symmetrical point is taken as

$$p_0 = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} x_1^{(i,j)}(\theta_2 x_2^{(i,j)} + \theta_3 x_3^{(i,j)} + \theta_4 x_4^{(i,j)})}{\sum_{i=1}^{M}\sum_{j=1}^{N} x_1^{(i,j)} x_1^{(i,j)}} \quad (14)$$

Based on the conditional posterior distribution we have obtained, we can sample all parameters in f_n.

4 Experiment Result

The experiments are performed using some Brodatz textures [11]. First, we estimate the parameters for one particular texture image with MCMC method, then synthesis texture with the parameters. The process of texture synthesis is the same as that sampling method used in Markov random field model.

Compared with sampling process of Markov random field model, the number of gray level for texture generated by this maximum entropy random field model suffers no limits. The two synthesis texture are generated with 256 gray levels. These two texture are learned from one fur texture and one vertical line texture, and the original pictures are showed in Figure 3 and synthesis textures showed in Figure 4.

5 Conclusion and Future Work

Based on the concept of maximum entropy and 2-D filter, we propose a new model to represent the texture in this paper. The model can be used to synthesis many types of texture. The parameters in the filter play an important role in forming the texture patterns. We propose a parameter estimation method based on MCMC approach in order to obtain the parameters in the filter. This method is much robust comparing with other traditional approaches.

Experiments show that maximum entropy random field model is capable of representing simple texture patterns, just like what Markov random field model can produce. However, it has only limited capability for natural texture description. The solution to this problem lies on adding more filters on this maximum random field model. The selection and estimation of the filter set are currently being studied.

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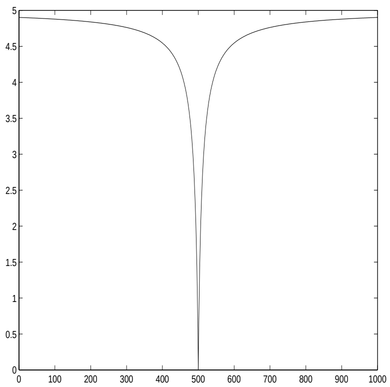

Figure 1: Potential function $\lambda()$

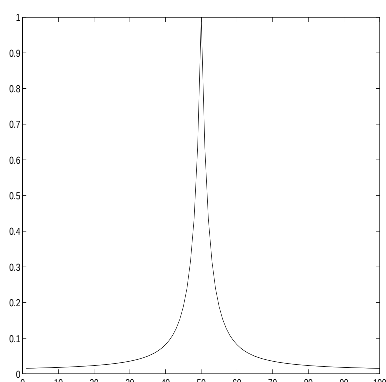

Figure 2: Typical pdf

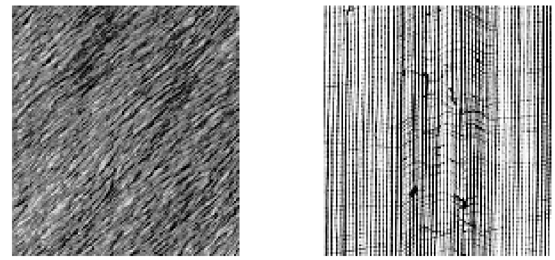

Figure 3: The original Texture images

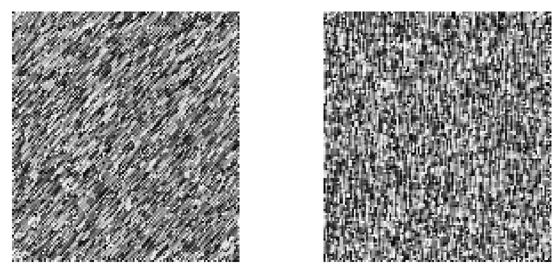

Figure 4: The synthesized texture images